

Why π Is Not 3.14: Understanding Approximation, Infinite Series, and Ramanujan's Formula

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Abstract

Pi, written as π , is one of the most famous numbers in mathematics. In school, students often use 3.14 or $22/7$ whenever a problem mentions π . These values are useful shortcuts, but they are not the same as π itself. This research paper explores the difference between an approximation and an exact mathematical constant. It begins with a simple calculus limit that shows why replacing π with $22/7$ can lead to the wrong answer. The paper then explains the meaning of π as the ratio of a circle's circumference to its diameter, and it discusses why π is irrational and transcendental. Next, it studies the Basel Problem, where Euler connected π to the infinite sum of reciprocal squares. Finally, the paper examines Srinivasa Ramanujan's famous infinite series for $1/\pi$, which shows how deeply π is connected to patterns, symmetry, and advanced number theory. The main argument is that π should not be treated as just 3.14. It is an exact, infinite, and powerful idea that connects geometry, calculus, number theory, and mathematical creativity.

Keywords: π , pi, approximation, irrational number, infinite series, Basel Problem, Ramanujan

Introduction

Most students first meet π in a geometry class. It appears in formulas like $C = 2\pi r$ for the circumference of a circle and $A = \pi r^2$ for the area of a circle. Because many school exercises use simple measurements, students are often told to take π as 3.14 or $22/7$. This makes calculations easier, especially when a calculator is not allowed. However, this shortcut can also create a misunderstanding. It can make students believe that π is actually equal to 3.14 or $22/7$. That is not true. Those values are only approximations.

This paper grew out of a small but important observation from the researcher's original notes: many students quickly replace π with $22/7$ even when a problem asks for an exact mathematical

answer. In the uploaded research draft, the student used a limit problem to show that this habit can be misleading. The expression $\lim_{x \rightarrow \pi} (x - 22/7)$ does not equal 0. The limit is $\pi - 22/7$. Since π and $22/7$ are not the same number, the difference between them is not zero.

This may seem like a small point, but it opens a much larger discussion. The number π is not only a tool for measuring circles. It appears in infinite series, probability, physics, engineering, wave motion, and many areas of higher mathematics. According to Encyclopedia Britannica, π is the ratio of a circle's circumference to its diameter, and common values such as 3.14 and $22/7$ are only approximations used for everyday calculations (Encyclopedia Britannica, 2026a). This means π has an exact identity even though its decimal digits never end.

The purpose of this research paper is to explain π in a way that a high school student can understand while still respecting its deeper mathematical importance. The paper is not trying to prove every advanced theorem about π . Instead, it explains the main ideas clearly: what π is, why 3.14 and $22/7$ are not exact, how a simple limit reveals the danger of shortcuts, how Euler found π inside an infinite series, and how Ramanujan discovered a formula that calculates π with amazing speed.

The central research question is: Why is it mathematically incorrect to treat π as 3.14 or $22/7$, and what do infinite series such as Euler's and Ramanujan's formulas reveal about the deeper nature of π ? The answer is that π is an exact constant with infinite depth. Approximations are useful, but they are not the full truth.

Background: What π Means

The most basic definition of π is simple: π is the ratio of a circle's circumference to its diameter. The circumference is the distance around the circle. The diameter is the distance across the circle through its center. No matter how large or small the circle is, the ratio between these two measurements is always the same. If a circle is huge, the circumference and diameter are both larger. If a circle is tiny, both are smaller. But the ratio remains π .

This is one reason π is so powerful. It does not depend on a unit like centimeters, inches, or meters. A person measuring a wheel, a plate, a coin, or the orbit of a planet will still meet the same constant. In this sense, π is not just a number on a page. It is a stable relationship found in every perfect circle.

However, there is a strange feature hidden inside this simple definition. A circle looks smooth, clear, and complete, but the number that describes it is endless. The decimal expansion of π begins 3.1415926535..., but it does not stop or settle into a repeating pattern. This is what makes π an irrational number. An irrational number cannot be written exactly as a fraction of two integers. Britannica explains that π is irrational and that approximations such as 3.14 or $22/7$ are used because the exact decimal expansion does not end (Encyclopedia Britannica, 2026a).

The fraction $22/7$ is especially interesting because it is very close to π . It equals approximately 3.142857. The actual value of π is approximately 3.141593 when rounded to six decimal places. These numbers are close, but close does not mean equal. In mathematics, exactness matters. A bridge can be close to straight and still not be perfectly straight. A clock can be close to correct and still be wrong by a few seconds. In the same way, $22/7$ can be close to π and still not be π . The number 3.14 is even less exact. It is a rounded decimal value that is convenient for basic school calculations. If someone wants to estimate the area of a circular table, using 3.14 may be enough. But if a problem asks for an exact answer, or if the calculation is part of advanced math, replacing π with 3.14 changes the meaning of the problem. It turns an exact constant into a rounded guess.

π is also called transcendental. This is more advanced than irrationality. A transcendental number is not the solution of any polynomial equation with rational coefficients. In simpler language, π cannot be fully captured by ordinary algebraic equations using whole-number powers and rational coefficients. Britannica defines transcendental numbers as numbers that are not algebraic, meaning they are not roots of polynomial equations with rational coefficients (Encyclopedia Britannica, 2026b). This property makes π even more unusual. It is not only impossible to write as a simple fraction; it is also beyond the reach of normal algebraic equations.

For a high school student, the most important idea is this: π is exact, but its decimal form is never complete. When we write 3.14, we are writing a useful approximation. When we write $22/7$, we are writing another approximation. But when we write π , we are writing the full constant in its exact form.

Methodology

This research paper uses a conceptual and explanatory method. It does not involve a laboratory experiment, a survey, or a statistical study. Instead, it studies a mathematical idea by comparing exact values, approximations, limits, and infinite series. This is appropriate because the research question is about mathematical meaning rather than human behavior.

The first part of the method is a worked example. The paper studies the limit $\lim_{x \rightarrow \pi} (x - 22/7)$. This example is important because it shows how a common classroom habit can create a wrong conclusion. By applying the basic idea of continuity, the expression can be evaluated by replacing x with the value it approaches, which is π . The result is $\pi - 22/7$, not 0. The example is small, but it clearly shows the difference between approximation and equality.

The second part of the method is a literature-based explanation. The paper uses reliable sources to explain the definition of π , irrational and transcendental numbers, Euler's Basel Problem, and Ramanujan's formula for $1/\pi$. These sources include encyclopedia entries, mathematical biographies, original mathematical writing, and modern papers that discuss Ramanujan's formulas.

The third part of the method is interpretation. After presenting the facts and formulas, the paper explains what they mean in simple language. This is important because many students see formulas as symbols to memorize. The goal here is different. The goal is to show that formulas can tell a story. The story of π begins with a circle, moves into infinite sums, and reaches some of the most beautiful patterns in mathematics.

The paper therefore combines three approaches: a classroom-style example, a historical review, and a mathematical explanation. Together, these approaches make π easier to understand while also showing that it is much deeper than the school-level value 3.14.

The Limit Example: Why the Difference Matters

The simplest way to understand the problem with approximating π is to look carefully at one expression:

$$\lim_{x \rightarrow \pi} (x - 22/7)$$

At first, the expression looks easy. Many students may think, " π is $22/7$, so x is approaching $22/7$, and the answer must be 0." This is exactly the kind of mistake this paper studies. The

mistake is not in the algebra. The mistake is in the assumption that $\pi = 22/7$. That assumption is false.

To solve the limit correctly, we need to remember what a limit means. The notation $x \rightarrow \pi$ means that x gets closer and closer to π . It does not mean x gets closer to $22/7$. The expression $x - 22/7$ is a continuous expression, which means there is no break or special problem when x reaches π . So we can substitute $x = \pi$ directly into the expression. This gives:

$$\pi - 22/7$$

This is the exact answer. If we want an approximate decimal, we can calculate it as about -0.001264 . The answer is small, but it is not zero. In mathematics, a small difference can still be important. This is especially true in calculus, where exact values and limits are used to build larger results.

This example teaches a larger lesson. Approximations are useful when a problem asks for a decimal estimate. They are not useful when the problem asks for exact reasoning. If a teacher says, "Use 3.14 for π ," then using 3.14 is acceptable because the instruction is clear. But if a mathematical expression contains π , and no approximation is requested, the exact symbol should be preserved.

The same idea appears in other parts of mathematics. For example, $\sqrt{2}$ is about 1.414, but $\sqrt{2}$ is not equal to 1.414. The fraction $1/3$ is about 0.333, but 0.333 is not exactly $1/3$. In each case, the decimal is helpful, but it loses information. The exact symbol holds the complete value.

Therefore, the limit example is not just a trick problem. It is a reminder that mathematical symbols have exact meanings. π is not a nickname for 3.14. It is an exact constant. $22/7$ is not another form of π . It is a nearby fraction. When students understand this, they begin to see mathematics as a language where accuracy matters.

π as an Approximation Problem

Approximating π has been an important challenge for thousands of years. Before modern calculators, people needed practical ways to estimate circular measurements. Builders, engineers, astronomers, and mathematicians all needed workable values. This is why approximations such as 3, 3.1, 3.14, and $22/7$ became useful. They made real-world calculation possible.

But approximation always involves a trade-off. The easier a number is to use, the less exact it may be. The value 3 is very easy to use, but it is not very accurate. The value 3.14 is more

accurate. The fraction $22/7$ is also close. More digits, such as 3.14159265, are even better. Still, no finite decimal can equal π exactly because π 's decimal expansion never ends.

A useful way to think about this is to imagine zooming in on a number line. If we mark 3.14, $22/7$, and π , the points are very close to each other. From far away, they may seem almost identical. But if we zoom in enough, we can see that they are separate points. Mathematics often works at this zoomed-in level. It cares about exact location, not just general closeness.

This explains why exact notation is so valuable. The symbol π represents the whole value, even though we cannot write every digit. It is similar to a person's name. A nickname may be useful, but it is not the full legal name. In the same way, 3.14 is a useful nickname for π in everyday calculations, but it is not the complete mathematical object.

There is also a difference between practical math and theoretical math. In practical math, approximations are often enough. A carpenter cutting a circular tabletop does not need millions of digits of π . Even scientists often use only a limited number of digits because measurement tools have physical limits. But in theoretical math, exactness matters because proofs depend on exact relationships. Proof cannot say something is true merely because it is close. It must show that it is exactly true.

This is why students should learn both sides. They should know how to use approximations when they are useful, but they should also know that approximations are not identities. 3.14 helps us calculate. π helps us understand.

Euler and the Basel Problem

One of the most surprising appearances of π happens in the Basel Problem. At first, this problem does not look like it has anything to do with circles. It asks for the exact value of the infinite sum:

$$1 + 1/4 + 1/9 + 1/16 + 1/25 + \dots$$

In sigma notation, this is written as:

$$\sum_{n=1}^{\infty} 1/n^2$$

This means we add the reciprocals of all square numbers forever. The terms get smaller and smaller, so the sum approaches a fixed number. But the question is: what exact number does it approach?

Leonhard Euler famously showed that the answer is:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

This result is amazing because the problem begins with whole numbers and fractions, not circles. There is no visible circle in the expression $1 + 1/4 + 1/9 + 1/16 + \dots$. Yet π appears in the final answer. The Basel Problem was originally posed in the seventeenth century and was solved by Euler in the eighteenth century; modern discussions of the problem still present it as one of the most celebrated results in mathematical analysis (Aste, 2021).

For a high school student, the exact proof may be too advanced, but the meaning can still be understood. The sum uses square numbers: $1^2, 2^2, 3^2, 4^2$, and so on. These are simple integer patterns. Euler's result shows that when this pattern is extended infinitely, it connects to π . This means π is not locked inside geometry. It also appears inside number patterns.

One explanation involves the sine function. The sine function is connected to circles because it comes from the unit circle in trigonometry. It also has an infinite series and an infinite product form. Euler compared these forms and found that the reciprocal square sum must equal $\pi^2/6$. The full reasoning requires advanced ideas, but the key point is accessible: infinite processes can reveal hidden connections between different areas of math.

The Basel Problem is important for this paper because it supports the main argument. If π were only a circle-measuring number, it would be surprising to find it in a sum of reciprocal squares. But if π is a deeper constant connected to cycles, waves, symmetry, and infinity, then its appearance makes more sense. It shows that π is a bridge. It connects geometry to analysis, finite numbers to infinite series, and simple patterns to advanced mathematics.

This also changes how students should think about π . In a basic classroom, π may seem like a number used only when a shape is round. Euler's result shows that this is too limited. π appears when mathematics studies infinite structure. The Basel Problem helps reveal π as a universal constant rather than just a geometry shortcut.

Ramanujan's Contribution to π

Srinivasa Ramanujan is one of the most remarkable figures in the history of mathematics. He was born in India in 1887 and developed many original ideas in number theory, continued fractions, infinite series, and other advanced areas. The MacTutor History of Mathematics

archive describes his work as making major contributions to the analytical theory of numbers, elliptic functions, continued fractions, and infinite series (O'Connor & Robertson, n.d.).

Ramanujan is especially important in this paper because he discovered powerful formulas for calculating π . His work shows that π is connected not only to circles and infinite sums, but also to deep patterns that are difficult even for many trained mathematicians. In 1914, Ramanujan published a paper titled "Modular Equations and Approximations to π ," where he presented extraordinary formulas related to π (Ramanujan, 1914).

One of his most famous formulas is:

$$1/\pi = (2\sqrt{2} / 9801) \sum_{k=0}^{\infty} [(4k)! (1103 + 26390k)] / [(k!)^4 396^{(4k)}]$$

This formula looks complicated, but its basic purpose can be explained simply. It gives a way to calculate $1/\pi$ using an infinite sum. Once $1/\pi$ is known, π can be found by taking the reciprocal.

The amazing part is how quickly the formula works. Each new term adds many correct digits of π . Modern papers continue to study and prove Ramanujan's formulas because of their beauty and depth (Ern & Wangsa, 2024; Guillera, 2018).

To understand why this is impressive, compare it with slower methods. Some infinite series approach π very slowly. A person may add many terms and still get only a few accurate digits.

Ramanujan's formula is different. It moves toward π extremely fast. This is why his work became important not only for theory but also for computing π to many digits. Borwein, Borwein, and Bailey (1989) discussed the relationship between Ramanujan's modular equations and high-speed methods for computing enormous numbers of digits of π .

Ramanujan's formulas are linked to modular equations and deep symmetries. For a high school audience, the exact theory is not necessary. What matters is the idea that Ramanujan found hidden patterns in numbers. His formulas suggest that mathematics has structures that are not obvious at first glance. A student may see π as a decimal. Ramanujan saw π as part of a much larger network of infinite relationships.

This is one reason Ramanujan inspires young researchers. He reminds students that mathematics is not only about following steps from a textbook. It is also about noticing patterns, asking questions, and trusting curiosity. Of course, modern mathematics also requires proof, checking, and clear explanation. But curiosity is often the starting point. Ramanujan's work shows how one person's unusual insight can open new paths for future mathematicians.

In the context of this research paper, Ramanujan's formula proves that π cannot be reduced to a school shortcut. A number that can be connected to such deep infinite patterns is much more than 3.14. It is a gateway into advanced mathematics.

Discussion: What π Teaches Students About Mathematics

The study of π teaches several lessons that go beyond one number. The first lesson is the difference between approximation and equality. In daily life, people often treat close values as good enough. This is practical and sometimes necessary. But mathematics has a stricter language. If two values are not exactly the same, they are not equal. This distinction is important in algebra, calculus, proof, computer science, engineering, and physics.

The second lesson is that simple ideas can lead to deep results. The definition of π begins with a circle, which is one of the simplest shapes. But from that shape, students can reach irrational numbers, transcendental numbers, infinite series, and formulas discovered by Ramanujan. This path shows that mathematics is layered. A concept may look simple at first, but deeper study can reveal surprising complexity.

The third lesson is that mistakes can lead to research. The limit example may begin as a classroom mistake: replacing π with $22/7$. Instead of simply saying the answer is wrong, a student can ask why it is wrong. That question becomes the beginning of a research project. In this way, confusion is not always a problem. It can be a doorway into better understanding.

The fourth lesson is that mathematics connects different areas. π begins in geometry, appears in trigonometry, enters calculus, shows up in infinite series, and becomes part of number theory.

The Basel Problem is a perfect example of this connection. It begins with fractions made from square numbers but ends with $\pi^2/6$. Ramanujan's formula continues this pattern by connecting π to modular equations and infinite sums.

The fifth lesson is that exact symbols are powerful. Writing π is not a failure to know the decimal value. It is actually a more exact way to write the number. This is important for students who sometimes think decimals are more "real" than symbols. In advanced math, symbolic form is often stronger because it keeps the full meaning. The expression $\pi - 22/7$ is more exact than a rounded decimal difference.

Finally, π teaches humility. Even though π is one of the oldest and most studied constants, it still inspires research. Mathematicians continue to explore its digits, formulas, computational

methods, and connections to other areas. For students, this is encouraging. It shows that mathematics is not finished. There are still patterns to discover and questions to ask.

Implications for High School Learning

High school mathematics often focuses on getting the correct answer quickly. This is understandable because students have tests, homework, and time limits. However, the story of π shows that students should also learn why answers are correct. If students only memorize formulas, they may miss the meaning behind them. If they understand why a formula works, they can apply it more carefully.

Teachers often introduce π through formulas for circles. This is a good beginning, but it should not be the ending. Students should also learn that 3.14 is an approximation. A short discussion of irrationality can make this clearer. Even younger students can understand that some decimals end, some repeat, and some continue forever without repeating. This helps them see why π cannot be fully written as a decimal.

Calculus students can learn an even stronger lesson from the limit example. When solving limits, they must pay attention to the exact value being approached. The expression $x \rightarrow \pi$ means x approaches π , not $22/7$. This teaches precision. It also shows why replacing constants too early can damage a solution.

Students interested in research can use π as a model for exploring math. A possible project could compare different approximations of π and calculate their errors. Another project could explain Euler's Basel Problem in simple language. A third project could study Ramanujan's formula and show how quickly it converges compared to a slower series. These projects would allow students to combine computation, history, and explanation.

The study of π also helps students build mathematical communication skills. It is not enough to write formulas. A researcher must explain what the formulas mean. This paper uses simple language because mathematical ideas become more powerful when more people can understand them. Making a difficult idea clear is not the same as making it weak. In fact, clear explanation often shows deeper understanding.

For a high school student, the most important takeaway is that curiosity matters. A small question like "Why is π not $22/7$?" can become a serious research topic. The goal is not to solve every mystery of π . The goal is to understand one idea deeply and explain it well.

Limitations

This paper has some limitations. First, it does not give full formal proofs of π 's irrationality or transcendence. These proofs are advanced and would require mathematical tools beyond the usual high school curriculum. Instead, the paper explains what those properties mean and why they matter.

Second, the paper does not fully prove Euler's solution to the Basel Problem. A complete proof would involve infinite products, trigonometric functions, or other advanced methods. The paper focuses on the meaning of the result rather than every technical step.

Third, the paper does not prove Ramanujan's formula for $1/\pi$. Modern proofs of Ramanujan-type series can involve hypergeometric series, modular forms, and algebraic number theory. These topics are far beyond the scope of a short high school research paper. However, the paper still presents the formula accurately and explains why it is important.

These limitations do not weaken the main argument. The goal of the paper is explanatory, not purely technical. It aims to make the deeper nature of π understandable to a general reader while using reliable mathematical sources.

Conclusion

π is often introduced as 3.14, but this paper has shown why that introduction is incomplete. The values 3.14 and $22/7$ are useful approximations, not exact identities. The difference matters because mathematics depends on precision. The limit $\lim_{x \rightarrow \pi} (x - 22/7)$ shows this clearly. The answer is not 0. It is $\pi - 22/7$, because π is not equal to $22/7$.

The paper then showed that π is much deeper than a circle formula. Its basic definition comes from the ratio of circumference to diameter, but its importance extends far beyond geometry. Because π is irrational, its decimal digits never end or repeat. Because it is transcendental, it cannot be captured by ordinary algebraic equations. These properties make π a special and exact constant.

Euler's solution to the Basel Problem shows that π appears in infinite sums of reciprocal squares. This connects π to number patterns and mathematical analysis. Ramanujan's formula for $1/\pi$ shows an even more surprising connection. His infinite series calculates π with extraordinary speed and reveals hidden structures in numbers.

The final lesson is simple: π should not be treated as a boring school shortcut. It is a symbol of how mathematics can grow from simple observation into infinite depth. A circle may look ordinary, but the number behind it is extraordinary. For students, π is an invitation to think more carefully, question shortcuts, and explore the patterns that make mathematics beautiful.

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